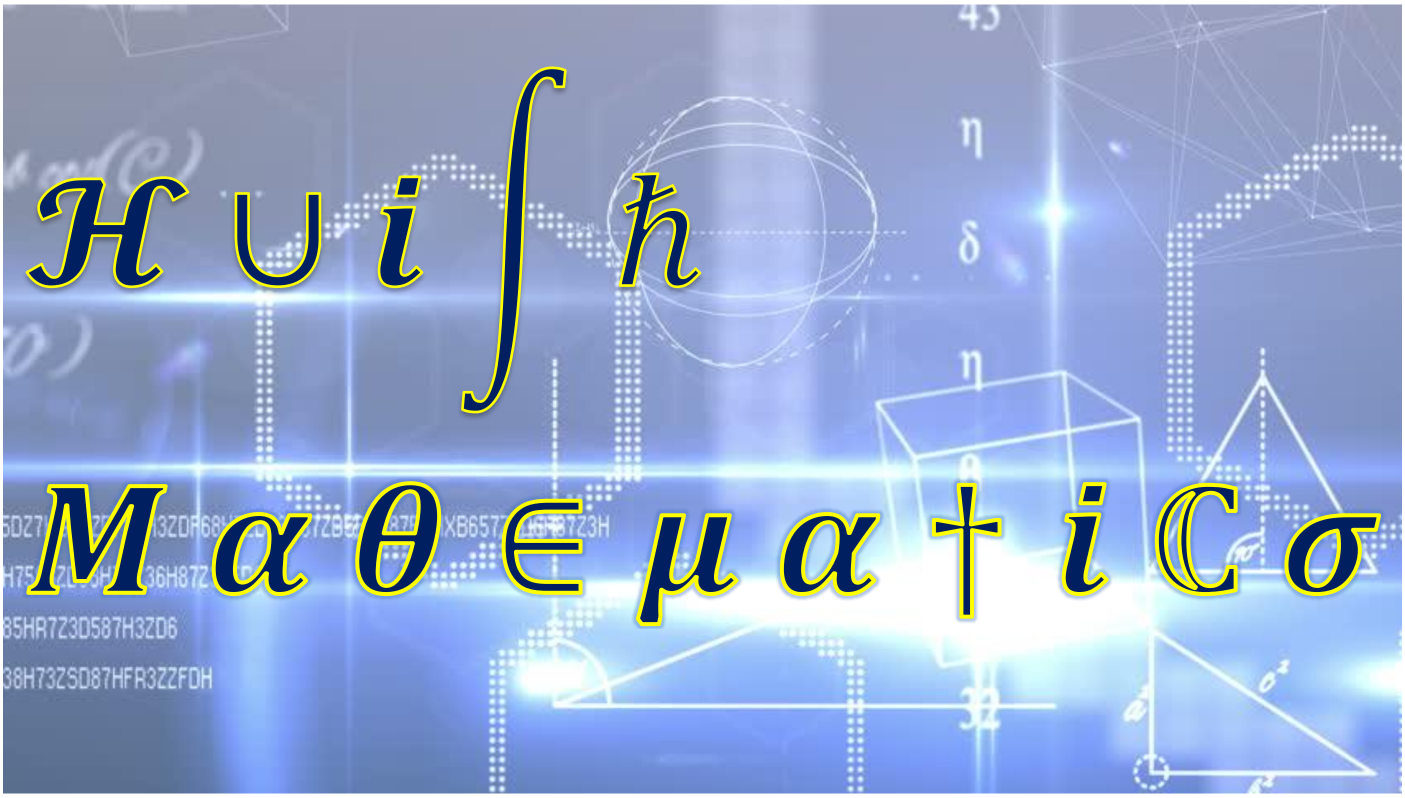


Preparing for A-level Mathematics

Welcome to Maths at Richard Huish!

Mathematics is one of the most widely taken A-level courses nationally, recognized by many for its "usefulness". However, many do not realize that it is a very important and fascinating subject in its own right, once the essential skills have been mastered.

A-level Maths, and especially A-level Further Maths, provide challenging experiences exploring this subject, and for the most successful, developing resilient problem-solving skills for future study and work.

Click here and here for some reasons for studying Mathematics and for choosing Maths as an A-level. (Though not *all* Economics degrees need Maths!)

FAQ

What is the difference between Maths and Further Maths?

The main difference is that Further Maths is a TWO A-level course, covering all of the A-level Maths course PLUS a whole extra A-level of Maths. This means having twice as many lessons and twice as much homework, so it is best for students who are pretty committed to studying Maths at university, or a very closely related subject like Engineering or Physics. Nationally, Further Maths is almost only taken by the strongest A-level Maths students, so we run AS Further Maths in the first year as a fourth subject, then if things go well, we can review your options at the end of year 1.

In Year 1, Further Maths students take part in A-level Maths lessons, and have extra lessons covering the AS Further Maths course. In Year 2, they leave their A-level Maths groups to join two A-level Further Maths classes.

The course content of Further Maths not only deepens the understanding and complexity of many topics met on the A-level Maths course, but also adds new areas, such as matrices.

How many lessons will I have each week?

At the time of writing, we expect that A-level Maths students will have 4 lessons each week, each lasting 1 hour and 5 minutes. Further Maths students will have an extra 4 lessons each week.

How much self-study/homework will I get each week?

Like all A-level course at Richard Huish, you should expect to do about 4 – 5 hours each week if you are doing A-level Maths. Those choosing Further Maths would expect to do another 4 – 5 hours each week. If you complete your work early, then Richard Huish students use the spare time to revise or look for extension work.

What help is available if I get stuck?

You will get stuck, and the Maths team recognize that this will happen. Once you have made a fair attempt to try and figure things out for yourself (this is what "resilience" in problem-solving looks like), your Maths teacher will be happy to help you out as best they can. Depending upon staffing levels, we may also run some Supervised Study sessions throughout the week that you could join – but at the time of writing things are uncertain.

What grades should I have?

The college's recommended minimum to study A-level Maths is a grade 6 at GCSE in Mathematics in addition to the standard college entrance requirements. For Further Maths it is a grade 7, with a general GCSE profile strong enough to support doing 4 A-levels in the first instance (which means having mainly grades 8 or 9 across most or all of your subjects). This is decided by a senior member of staff in each case when joining us in September.

What do I need to succeed at A-level Maths?

The following characteristics are those that we see in all our most successful candidates:

- Excellent algebraic skills.
- A logical, organized and determined approach to problem solving.
- Resilience to try different ideas, and to think things through for themselves – Maths cannot be done by simply following "stock examples" at A-level. Mathematicians must understand the meaning of the ideas that have been taught, so that they can use them in original ways to tackle question unlike anything they have seen before.
- An understanding that in Mathematics, all things must be logically reasoned and explained to be reliable.

What will I study?

At A-level, there are three over-arching “themes” that run through all of the topics we study. In short, they are

- Proof
- Modelling
- Problem solving

You will have met proof a bit on the GCSE course; the role of proof is much greater at A-level (and in higher level Mathematics, it becomes the main focus of the subject!)

Modelling refers to using mathematical functions and methods describe in general terms how real systems behave (including how viruses spread)

Problem solving means figuring out what mathematics to use and how to use it when you meet a question of a type which you haven’t seen before; mathematicians use logic, and the knowledge of mathematical techniques to break down and analyse difficult questions and use mathematics as a “tool” to tackle problems.

In Year 1, Mathematics students study the following topics:

- Proof
- Algebra and functions
- Graphs of functions, including their transformations (translations, reflections and stretches)
- Coordinate geometry
- Trigonometry and trigonometric functions
- Logarithms and exponential functions
- Differentiation (or “differential calculus”)
- Integration (or “integral calculus”)
- Kinematics
- Vectors
- Simple dynamics
- Statistical sampling
- Data presentation and interpretation
- Probability

In Year 1, Further Maths students also study the following topics:

- Complex numbers
- Matrices
- Further algebra and functions
- Further calculus
- Further vectors
- Polar coordinates
- Hyperbolic functions

What resources will I need?

At the time of writing, we use two online textbooks which include extra features beyond the printed versions. After joining the college, you will be issued with your own free personal login details for each text book giving you access to all the student resources associated with each text.

We use these resources both in class and when setting your self-study work. Ideally, you would have the use of a device that gives access to the internet both in college and at home – most of our students use their smart phones, whilst some use a tablet (which has a bigger screen of course) or a small laptop. If you do not have any such device, the department has a small number of laptops that can be borrowed during each lesson, and the college also has laptops available for short term loans.

You will also need to obtain an "Advanced Scientific Calculator" after the first term of the course, but we strongly advise that you wait until the end of the first term before purchasing a new calculator; the one you have used for GCSE will be sufficient at the start of the course.

You will also need:

- A folder with dividers
- A4 lined and punched paper
- Black pens and pencils
- Ruler

Other materials such as blank revision cards can be purchased after the start of the course, but you will need to use them too.

What's in this Pack?

The rest of this "Huish Headstart" Maths pack mainly contains resources, exercises and activities that focus on those parts of the GCSE Higher Tier syllabus that you will need to have mastered before the start of the A-level course.

We know that some students may have missed out on one or two GCSE topics, so we have focussed on those parts of GCSE that are most important for A-level study, so that you don't waste your time on those areas of GCSE that you will not use so much. These topics have also been chosen because they will give you a better "feel" for what the nature of the A-level course is like; different students like different topics at GCSE – but it is *these* ones that you need to have liked the most (or at least, liked quite a lot)!

Acknowledgements

We'd like to thank the owners/authors of the following websites for the many resources linked in this booklet:

https://corbettmaths.com/
https://www.mathsgenie.co.uk/
https://www.examsolutions.net/
https://www.geogebra.org/
https://ed.ted.com/
https://nrich.maths.org/

How to use this booklet

There are six sections for you to work through, each covering one of the six topics listed below.

1. For each section, follow either the blue route or the green route from the Start button to the "Now try the questions overleaf" button.

   The blue route (down the left of the page) is your "Motorway to Mastery" – a bit like the M5 without the traffic jams. If you can follow the videos and examples on this route, then the blue route through the topic will usually be quicker.

   The green route (on the right) is a slower more "scenic" route. It has more videos and resources or examples that are arranged in smaller steps than the blue route. If you are not sure about a topic, or you quickly get stuck on the "Motorway", then use the green route.

   Whichever route you follow for each topic, follow the arrows until you reach the end point.

2. In sections 2 to 6 there are also purple boxes with some **optional** extra activities or resources. You do **not** have to do those, but we hope that you will find some of them interesting. We *would* expect most students thinking about doing Further Maths to try at least most of these optional activities.

3. In *some* sections there are also some printed notes on the topic and a small number of examples to provide you with more help with those topics.

4. **Before moving to the next topic, do the selection of GCSE questions at the end of the section**. Numerical answers are provided for you to check your work as you go along.

   If you get stuck, then your first strategy should be to go back and re-read or re-watch some of the resources at the start of the section.

   If that does not help get you go forward then we hope to provide an online query service where you can email us with your question. Please try to explain the difficulty you are having as clearly as you can. Once the system is set up, the email address should be maths@richuish.ac.uk. If that does not work, please email the Mathematics Course Manager, Jon Middle at jonm@richuish.ac.uk. We will be checking it several times a week

5. **At the very end of the booklet there is a "final assignment". All students are expected to complete this and before the start of classes and to bring it in for feedback. Again, online assistance will be available if needed for that.**

Topic 1: Factorising Quadratics

Factorising algebraic expressions is so frequent in A-level Mathematics, that you will need to be very confident about doing it. The online videos linked below show you some common methods for doing this, though there are others.

Mathematicians use "by inspection" – which is "Maths-speak" for "Guess until you get it right, and then check that it *is* right!"

Start

Factorising Quadratics (all forms)

Maths Genie lessons on factorising "trinomial" (3 term) quadratic expressions

By the "sum/product" or "grouping" method

By the "difference of two squares" formula $x^2 - y^2 = (x+y)(x-y)$

By "inspection" (the "professionals" way!)

Not sure? Maths Genie lessons on:

Expanding one bracket

Factorising with Common Factors (1 bracket)

Now try these online factorisation questions

Revise the "difference of two squares" formula $x^2 - y^2 = (x+y)(x-y)$ with this Corbett Maths video

Revise expanding and factorising two brackets

Corbett Maths: Expanding Brackets

Factorising "trinomials" like $1x^2 + bx + c$

Revise factorising harder "trinomials" like $ax^2 + bx + c$ where $a \neq 1$

"Splitting the middle term" method (Corbett)

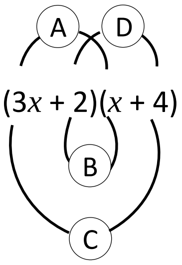
Factorising Quadratics (all forms)

Try these factorising questions from Maths Genie

Now try the questions overleaf!

Some further tips on how to figure out the factors of a "trinomial" quadratic (but don't forget common factors and difference of two squares for two term quadratics!)

Multiplication of brackets is carried out as follows:



(A) (B) (C) (D)

$= 3x \times x + 2 \times x + 3x \times 4 + 2 \times 4$

$= 3x^2 + 2x + 12x + 8$

$= 3x^2 + 14x + 8$

8 is the constant term and comes from the "product" (i.e. multiplication) of the last terms in each bracket (D)

The $3x^2$ term comes from the product of the **first terms in each bracket** (A)

The $14x$ term bracket (D) comes from the product of the **outer terms plus the product of the inner terms** [(B) and (C)]

To factorise $3x^2 + 14x + 8$ into two brackets, first note that:

$+3x^2$ comes from the product (i.e. multiplication) of the **first terms in each bracket** (A)

$+14x$ comes from the product of the **outer terms plus the product of the inner terms** [(B) and (C)]

+8 is the product of the **last terms in each bracke**t (D)

Also note that:

- If the x term is positive and the constant term is positive, then both brackets will contain "+"
- If the x term is negative but the constant term is positive then both brackets will contain "–".
- Finally if the constant term is negative, regardless of the x term the brackets will contain one "+" and one "–".

Now try these:

If you can print these pages off, then you can write your solutions in the spaces provided. If you cannot print them out, just copy each question on a piece of ordinary paper and write your solution neatly below it. Good luck.

Factorise fully each of these quadratics. (By factorise fully, we mean that you take out the highest common factors, both numerical and algebraic).

1. $18x^2 - 24x$
2. $x^2 + 3x - 28$
3. $x^2 - 5x - 24$
4. $9x^2 - 64$
5. $2x^2 - 7x - 15$
6. $12x^2 + 17x - 5$
7. $3x^2 + 16x - 35$
8. $20x^2 - 3x - 2$

Answers:

1. $6x(3x - 4)$ 2. $(x + 7)(x - 4)$ 3. $(x - 8)(x + 3)$ 4. $(3x + 8)(3x - 8)$

5. $(2x + 3)(x - 5)$ 6. $(3x + 5)(4x - 1)$ 7. $(3x - 5)(x + 7)$ 8. $(4x + 1)(5x - 2)$

Topic 2: Algebraic Manipulation: Changing the subject and algebraic fractions

Rearranging algebraic expressions is *the* basic skill used by all mathematics students at A-level to complete a solution to any problem, or to prove any formula or result that has to be used.

The following resources give examples of the kinds of GCSE techniques that you will be using frequently from the very start of the course. You will add new techniques to these as the course progresses.

Start

Revise Rearranging Formulae

Maths Genie Lesson Changing the Subject of a Formula (Higher)

Not sure?

Watch this Maths Genie Lesson on Changing the Subject of a Formula (Foundation)

Here are some more examples from Corbett Maths

Revise Simplifying Algebraic Fractions

Exam Solutions lesson on simplifying algebraic fractions by factorising first and cancelling

Maths Genie video on how to add or subtract algebraic fractions using a common denominator

Now try these algebraic fractions questions from Maths Genie

Work each on out first, then check your answer.

Not sure? Start with simplifying some numerical examples

Corbett Maths lesson on simplifying numerical fractions by dividing top and bottom by a common factor

Maths Genie video lesson on simplifying algebraic fractions

Now see how adding algebraic fractions is similar to adding numerical fractions.

Video showing how to add two fractions with different denominators

Maths Genie video lesson on simplifying algebraic fractions

Now try the questions overleaf!

Optional Extension on History of Algebra

Click

Click

Now try these:

1. Rearrange each of these formulae to make x the subject

   a) $5x - 12y = -6$

   b) $v^2 = u^2 + 2ax$

   c) $x + u = vx + 3$ (Hint: Get x terms on one side and factorise)

   d) $5(x - 3m) = 2nx - 4$ (Hint: Multiply out bracket, get x terms on one side and factorise)

   e) $(1 - 3x)^2 = t$ (Hint: Square root both sides first – remember to write $\pm\sqrt{\ldots}$!)

2. (a) Simplify

$$\frac{x^2 - 16}{2x^2 - 5x - 12}$$

   (b) Make v the subject of the formula

$$w = \frac{15(t - 2v)}{v}$$

3. Simplify fully

$$\frac{3x^2 - 8x - 3}{2x^2 - 6x}$$

4. Solve

$$\frac{x}{x+4} + \frac{7}{x-2} = 1$$

   You **must** show your working.

Answers.

Note that there are often several possible ways to write a correct answer. Some equivalent options are shown here:

1 a) $x = \frac{12y-6}{5}$ or $x = \frac{1}{5}(12y - 6)$ b) $x = \frac{v^2-u^2}{2a}$ or $x = \frac{v^2}{2a} - \frac{u^2}{2a}$ c) $x = \frac{3-u}{v-1}$ or $x = \frac{u-3}{1-v}$

d) $x = \frac{15m-4}{5-2n}$ or $x = \frac{4-15m}{2n-5}$ e) $x = \frac{-1\pm\sqrt{t}}{-3}$ or $x = \frac{1\pm\sqrt{t}}{3}$ (Other possible answers exist)

2 a) $\frac{x+4}{2x+3}$ b) $v = \frac{15t}{w+30}$ 3 $\frac{3x+1}{2x}$ 4. $x = -12$

Topic 3: Coordinate geometry of straight lines

There are many types of graph that you will study, sketch and interpret during the A-level Maths course. The most basic is that of the straight line. At GCSE its equation is usually in the form $y = mx + c$ where m is the gradient and c the intercept on the y axis.

At A-level, we frequently rearrange this into other, equivalent forms like $ax + by = c$ where a, b and c are constant ("fixed") numbers. You will need to be able to rearrange this into $y = mx + c$ whenever necessary. You will also need to be able to find the equation of a straight line given either

- two points on the line, or
- one point (NOT necessarily the y intercept!) and the gradient of the line.

Other knowledge you will also need include:

- recognising when two lines are parallel or perpendicular from their gradients alone,
- finding the distance between two points
- finding the coordinates of the midpoint of a line (segment).

Start

Revise the equation of a line given the gradient and any point with Exam Solutions here

More from Exam Solutions here

Optional activity: challenge yourself by generating your own straight line problems with Geogebra here

(Click Self Test to test; Learn to learn with Show Steps. Pick "whole number points for convenience!)

Not sure? Remind yourself of what the gradient means here with Exam Solutions

Here are some practice questions if you want them

Explore the effect and meaning of "m" and "c" in the $y = mx + c$ with this Geogebra app

Move the blue points around and change the gradient and intercept to fit the line through the points in this Geogebra app

Learn how to find the equation $y = mx + c$ given any two points with Corbett Maths here

How to find the equation $y = mx + c$ given the gradient and any one point with Exam Solutions

Recognising and using Parallel lines: Examples from Corbett Maths

Recognising and using Perpendicular lines: Explanation and examples from Corbett Maths

Check your knowledge with this quiz from Bitesize

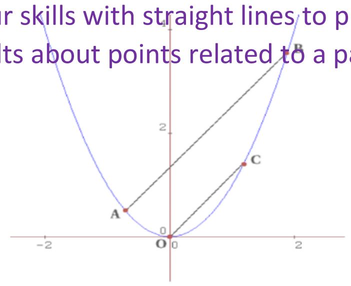
Revise how to:

Use Pythagoras's Theorem to find the distance between two points with Exam Solutions here

Find the coordinates of the midpoint of a line here with Exam Solutions

Optional Extension Activity

Use your skills with straight lines to prove these results about points related to a parabola



Now try the questions overleaf!

Now try these questions

1. Find the gradient of the line through each pair of points:

a) $(5, 2)$ and $(-4, -6)$ b) $(3a, a)$ and $(a, 5a)$

2. Calculate the exact* distance between each pair of points.

(*Exact means leave your answer in surd form)

a) $(-3, 9)$ and $(12, -7)$ b) $(k, -3k)$ and $(2k, -6k)$

3. Find the coordinates of the midpoint of each pair of points:

a) $(2, -4)$ and $(-3, -9)$ b) $(m, 2n)$ and $(3m, -2n)$

4. Work out the gradient and the y-intercept of these lines:

a) $y + 2x = 3$ b) $3(y - 2) = 4(x - 1)$

5. Find the equation of the line through each pair of points:

a) $(2, 5)$ and $(0, 6)$ b) $(-3, -7)$ and $(5, 9)$

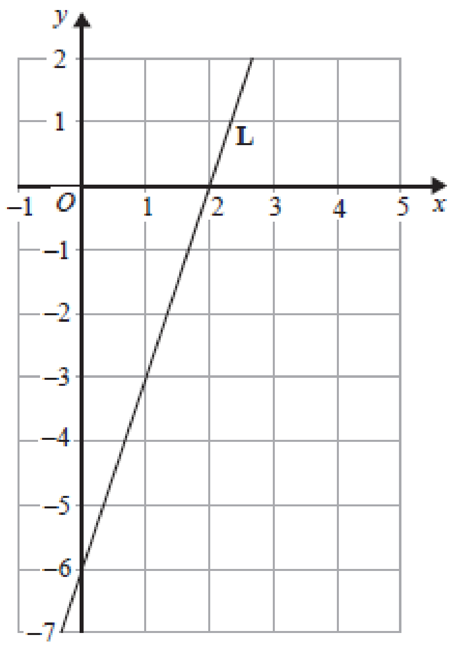
6. The line l_1 has equation $y = 5x + 1$

a) Find the equation of the line parallel to l_1 and which passes through the point $(3, -3)$

b) Find the equation of the line perpendicular to l_1 and which passes through the point $(-4, 1)$

7. The line L is shown on the grid.

Find an equation for L.



8. A is the point with coordinates $(5, 9)$. B is the point with coordinates $(d, 15)$.

The gradient of the line AB is 3. Work out the value of d.

Answers.

All answers are given in the simplest ("cancelled down") exact form. Decimals are **only** given as an alternative, where they are exact equivalent of the fractional value.

1 a) $\frac{8}{9}$ b) -2 2 a) $\sqrt{481}$ b) $\sqrt{10}\,k$ 3 a) $\left(-\frac{1}{2}, -\frac{13}{2}\right)$ or $(-0.5, -6.5)$ b) $(2m, 0)$

4 a) Gradient is -2 y-intercept is 3 b) Gradient is $\frac{4}{3}$ y-intercept is $\frac{2}{3}$

5 a) $y = -\frac{1}{2}x + 6$ b) $y = 2x - 1$

6 a) $y + 3 = 5(x - 3)$ or $y = 5x - 18$ b) $y - 1 = -\frac{1}{5}(x + 4)$ or $y = -\frac{1}{5}x + \frac{1}{5}$

7. $y = 3x - 6$ 8. $d = 7$

Topic 4: Solving Quadratic Equations

Quadratic equations are used very widely throughout the A-level Maths course from the very beginning. You could be asked to solve them, sketch (*not* plot) their graphs or to "complete the square" and to use that form to find the maximum or minimum. Follow this map to revise most of these GCSE methods.

Start

Revise solving a quadratic equation by factorisation.
Exam Solutions: Solution by factorising

Not sure?
A shorter video from Corbett Maths showing how to solve a quadratic by factorisation

Take this Geogebra quiz on solving quadratics by factorisation

Revise solving quadratics using the quadratic formula with this Exam Solutions video

Optional self-test questions on solving a quadratic using the formula

Revise how to write
$ax^2 + bx + c$ as $a(x + p)^2 + q$ where $a = 1$
"Completing the Square" with Exam Solutions

Optional: Revise how to write
$ax^2 + bx + c$ as $a(x + p)^2 + q$ where $a \neq 1$
Exam Solutions Video

Optional self-test questions on Completing the Square from Maths Genie

Optional: Learn how to **prove** the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ using the "Completed Square" form here

Try some of these self-test questions on Solving Quadratics from BBC Bitesize

Optional Extension
Explore what happens when a quadratic has no real roots, and extending the number system with these articles from NRICH.

Now try the questions overleaf!

Note that to solve any quadratic, you **<u>must</u> <u>first</u>** begin by re-writing it in the form:

$$ax^2 + bx + c = 0$$

(i.e. "…= 0") where a, b and c are some fixed numbers, and a is not zero.

Note:

- If $b = 0$, then the equation looks like $ax^2 + c = 0$. If c is negative, use "difference of two squares".

- If $c = 0$ the equation looks like $ax^2 + bx = 0$. These can be solved using "common factorization" like this:

$$ax^2 + bx = 0$$

$$x(ax + b) = 0$$

There are two methods of solving quadratics at GCSE which we use extensively at A-level:

- Factorization
  Write $ax^2 + bx + c = 0$ as two brackets or "factors" $(kx + m)(px + q) = 0$, and setting each bracket equal to zero to solve.

$$(kx + m)(px + q) = 0$$

$$kx + m = 0 \quad \text{or} \quad px + q = 0$$

$$kx = -m \quad \text{or} \quad px = -q$$

$$x = -\frac{m}{k} \quad \text{or} \quad x = -\frac{q}{p}$$

  If the quadratic is a simpler case where $c = 0$, we get common factorization, and just one "bracket"

$$x(ax + b) = 0$$

$$x = 0 \quad \text{or} \quad ax + b = 0$$

$$x = 0 \quad \text{or} \quad x = -\frac{b}{a}$$

- Using the formula
  If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Although many calculators will solve quadratics, **you are still expected to be able to do it "by hand"**. The skill of "factorizing" in the first method above is especially useful, and A-level Maths students do this very quickly.

Now try these questions

1. $x^2 + 7x + 12 = 0$
2. $3x^2 + 5x - 2 = 0$
3. $x^2 - 2x + 1 = 0$
4. $3x^2 + 5x + 2 = 0$
5. $x^2 - 1 = 0$
6. $x^2 - 2x - 3 = 0$
7. $8x^2 + 14x + 3 = 0$
8. $x^2 - 4 = 0$
9. $4x^2 - 9 = 0$
10. $12x^2 - 5x = 3$

Please solve the following quadratics using the formula, giving your answer in simplest surd form:

11. $x^2 - 3x - 2 = 0$
12. $4x^2 - 4x - 1 = 0$
13. $3x^2 + 2x - 7 = 0$
14. $7x^2 + 9x + 1 = 0$

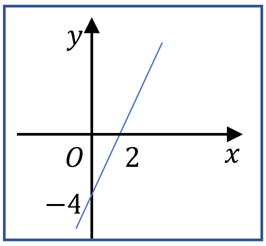
Answers

1. $(x + 4)(x + 3) = 0$; $x = -4$ or -3
2. $(3x - 1)(x + 2) = 0$; $x = \frac{1}{3}$ or -2
3. $(x - 1)(x - 1) = 0$; $x = 1$ (or 1)
4. $(3x + 2)(x + 1) = 0$; $x = \frac{-2}{3}$ or -1
5. $(x + 1)(x - 1) = 0$; $x = -1$ or $+1$
6. $(x + 1)(x - 3) = 0$; $x = -1$ or 3
7. $(4x + 1)(2x + 3) = 0$; $x = \frac{-1}{4}$ or $\frac{-3}{2}$
8. $(x + 2)(x - 2) = 0$; $x = -2$ or $+2$
9. $(2x + 3)(2x - 3) = 0$; $\frac{-3}{2}$ or $\frac{3}{2}$
10. $(3x + 1)(4x - 3) = 0$; $x = \frac{-1}{3}$ or $\frac{3}{4}$
11. $\frac{-(-3)\pm\sqrt{3^2 - 4\times1\times(-2)}}{2\times1} = \frac{3\pm\sqrt{17}}{2} \left(= \frac{3}{2} + \frac{\sqrt{17}}{2} \ \ or \ \frac{3}{2} - \frac{\sqrt{17}}{2}\right)$
12. $\frac{-(-4)\pm\sqrt{(-4)^2 - 4\times4\times1}}{2\times4} = \frac{4\pm\sqrt{32}}{8} = \frac{4\pm4\sqrt{2}}{8} = \frac{1\pm1\sqrt{2}}{2} \left(= \frac{1}{2} + \frac{\sqrt{2}}{2} \ \ or \ \frac{1}{2} - \frac{\sqrt{2}}{2}\right)$
13. $\frac{-2\pm\sqrt{2^2 - 4\times3\times(-7)}}{2\times3} = \frac{-2\pm\sqrt{88}}{6} = \frac{-2\pm2\sqrt{22}}{6} = \frac{-1\pm1\sqrt{22}}{3}\left(= \frac{-1}{3} + \frac{\sqrt{22}}{3} \ \ or \ \frac{-1}{3} - \frac{\sqrt{22}}{3}\right)$
14. $\frac{-9\pm\sqrt{9^2 - 4\times7\times1}}{2\times7} = \frac{-9\pm\sqrt{53}}{14} \left(= \frac{-9}{14} + \frac{\sqrt{53}}{14} \ \ or \ \frac{-9}{14} - \frac{\sqrt{53}}{14}\right)$

Topic 5: Algebra and Graphs

Look at these two objects: Surprisingly, mathematicians see these two objects as the *same thing*!

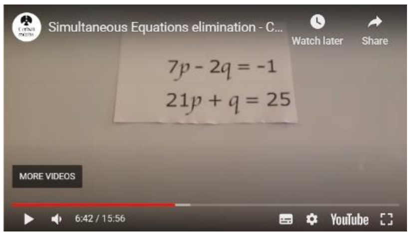
$$y = 2x - 4$$



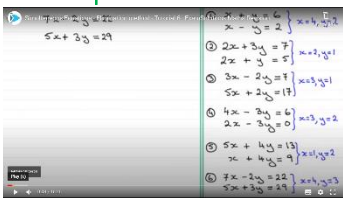
The equation, if "understood" correctly, describes the straight-line graph. This means that a great many problems in coordinate geometry can be solved using the algebraic equivalent. This is what you do when solving simultaneous equations at GCSE.

Start

Click the picture to revise solving two linear equations by elimination



Not sure?
Click below for a selection of solving two linear simultaneous equations from Exam Solutions

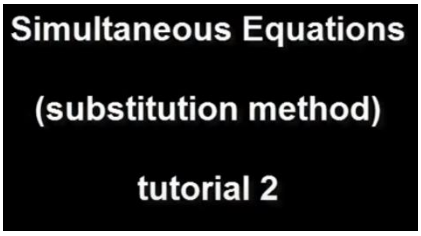


Try these self test questions on Maths Genie

Click below to revise how to find the points of intersection of a line and a circle

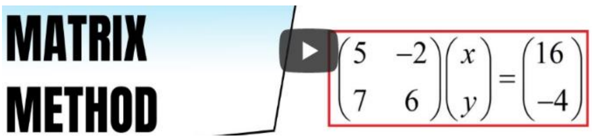
$$x^2 + y^2 = 5$$
$$x + y = 3$$

Click below to revise solving simultaneous equations with other non-linear equations



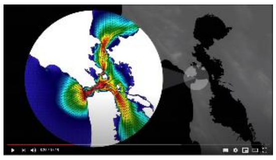
Try some simultaneous self-test questions from Maths Genie

Click below to learn how to code systems of linear simultaneous equations in matrix form.



Extra challenge: Write a spreadsheet to solve any pair of non-parallel linear equations

Watch this TED short talk showing how algebra lies behind mathematical modelling



Now try the questions overleaf!

For two linear equations, there are two methods using algebra that can be used:

Method 1: ("Elimination") and Method 2: ("Substitution")

E.g. Solve $x + 3y = 11$ (equation 1)
and $4x - 7y = 6$ (equation 2)

Method 1:

(equation 1) × 7:

$$7x + 21y = 77 \quad (3)$$

(equation 2) × 3:

$$12x - 21y = 18 \quad (4)$$

(eq. 3 + eq. 4): $19x = 95$

$$x = \frac{95}{19} = 5$$

Now find y:

(equation 1) $5 + 3y = 11$

$$3y = 6$$

$$y = 2$$

Method 2:

Rearrange equation 1 to make x the "subject":

$$x = 11 - 3y$$ (subtract $3y$ from both sides)

Substitute ("replace") x in equation 2 with $11 - 3y$:

$4(11 - 3y) - 7y = 6$ (Note use of brackets!)

$44 - 12y - 7y = 6$ (Expand the bracket)

$44 - 19y = 6$

$38 = 19y$ (Subtract 6, add $19y$)

$\frac{38}{19} = 2 = y$

Now find x:

$$x = 11 - 3y = 11 - 3 \times 2 = 11 - 6 = 5$$

Now try these:

Solve the following pairs of linear simultaneous equations, giving your answers in their simplest form where necessary. You may use either method, but must write out full workings:

1. $x + 3y = 11$
   $4x - 7y = 6$

2. $5x + y = 9$
   $3x + 4y = 2$

3. $3x - 2y = -2$
   $5x + y = 27$

4. $5x + y = 7$
   $2x + 3y = -5$

Answers:

1. $x = 2, y = 2$ 2. $x = 2, y = -1$ 3. $x = 4, y = 7$ 4. $x = 2, y = -3$

For one linear equation, and one quadratic, you MUST use substitution. Rearrange the LINEAR equation to make either x or y the subject, then substitute (i.e. “replace”) it in the quadratic equation.

Example:

Solve $3x + y = 8$ and $3x^2 + y^2 = 28$

Rearrange the linear equation (the first one here), to make y the subject. (It is easier to make y the subject):

$3x + y = 8 \quad \Rightarrow \quad y = 8 - 3x$ (subtract $3x$ from both sides)

Substitute for y in the second quadratic equation. **<u>Use brackets</u>**:

$3x^2 + y^2 = 28 \quad \Rightarrow \quad 3x^2 + (8 - 3x)^2 = 28$

Expand the bracket carefully. Remember that $(8 - 3x)^2 = (8 - 3x)(8 - 3x) = 8^2 - 24x - 24x + 9x^2$:

$$3x^2 + (8^2 - 24x - 24x + 9x^2) = 28$$

$$3x^2 + (64 - 48x + 9x^2) = 28$$

$$3x^2 + 64 - 48x + 9x^2 = 28$$

$$12x^2 - 48x + 36 = 0$$

It’s a LOT easier to divide both sides by the highest common factor of 12 in these terms:

$$x^2 - 4x + 3 = 0$$

Use factorisation or the formula to solve:

$$(x - 3)(x - 1) = 0$$

So $x - 3 = 0 \quad \Rightarrow \quad x = 3$ or $x - 1 = 0 \quad \Rightarrow \quad x = 1$

Now use $y = 8 - 3x$ to find the y values:

When $x = 3$, $y = 8 - 3x = 8 - 3 \times 3 = -1$ When $x = 1$, $y = 8 - 3x = 8 - 3 \times 1 = 5$

<u>Now try these:</u>

1. Solve $y - 4x = -1$ and $2x^2 - 3x + y = 2$
2. Solve $x + y = 3$ and $x^2 - 3y = 1$
3. Solve $y - x = 1$ and $2x^2 - 11y = -16$
4. Solve $y - x = 10$ and $x^2 + y^2 = 50$

Answers

1. $x = 1, y = 3$ and $x = -\frac{3}{2}, y = -7$
2. $x = -5, y = 8$ and $x = 2, y = 1$
3. $x = \frac{1}{2}, y = \frac{3}{2}$ and $x = 5, y = 6$
4. $x = -5, y = 5$ only

You will also need to make sketches of the graphs wide range of different functions (we do <u>not</u> *plot* graphs at A-level usually!).

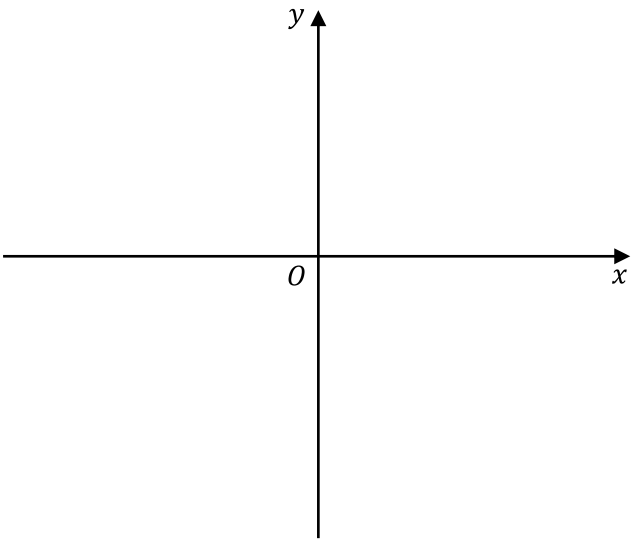
This is a very useful skill, for example when interpreting models in Mathematics. Revise the common shapes of graphs that you will have met at GCSE below. Either print this page or just draw four sets of neat axes on ordinary paper.

Each pair of axes has an equation. Use the Geogebra links above each pair of axes to make a sketch of the general shape; be sure to mark **<u>clearly</u>** on your axes any points of interest including:

- y-intercepts (mark this on the axis)
- x-intercepts (mark this on the axis)
- the general shape of the graph (make a neat, clear sketch)
- maximum or minimum points (mark these on the graph where they exist)

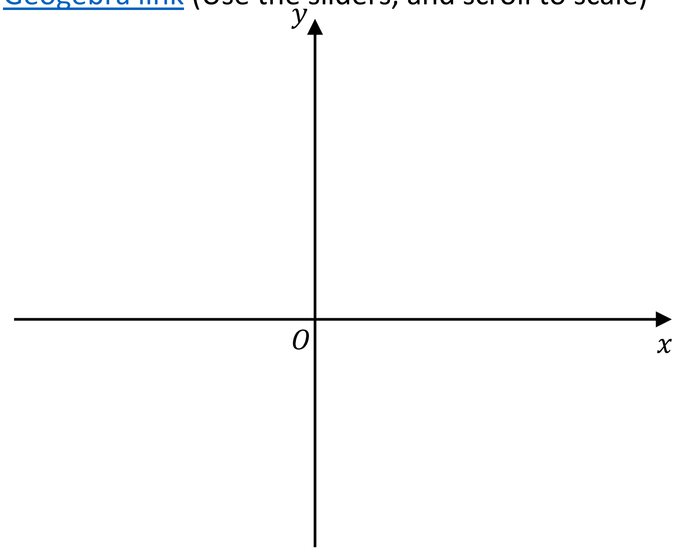
$$y = (x-2)^2 - 5 = x^2 - 4x - 1$$

Geogebra link (Pick which form and use the sliders)



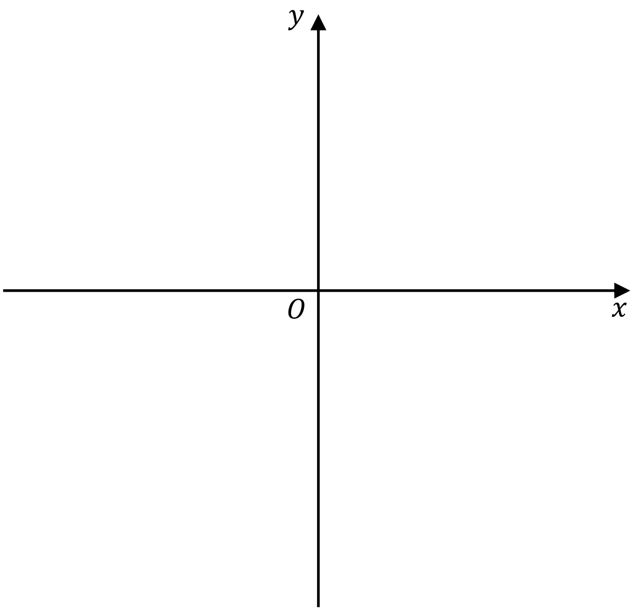
$$y = (x+3)(x-1)(x-4) = x^3 - 2x - 11x + 12$$

Geogebra link (Use the sliders, and scroll to scale)



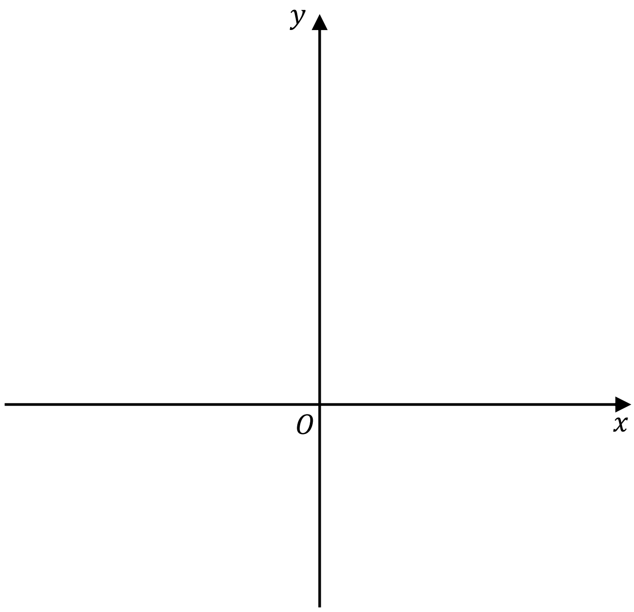
$$y = \frac{1}{x} \quad (x \neq 0)$$

Geogebra link (set a=1, h=0, k=0 first for the sketch)



$$y = 2^x$$

Geogebra link



Topic 6: Indices/Exponents

At A-level, you will have to handle a wide range of expressions involving indices (also known as "powers"). You will also need to be able to use the laws of indices to simplify expressions.

Start

A recap of all the main laws of indices, including the meaning of negative and fractional powers with Exam Solutions here

Not sure?
Recap positive integer indices here
Recap the multiplication law of indices here
Recap the division law of indices here
Recap negative integer indices here
Recap the "power to a power" law here
Recap fractional indices here

Now try this test from BBC Bitesize

Explore using the laws of indices at 3 different levels with this Geogebra quiz.

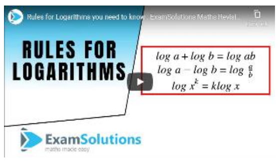
Now practice using the laws of indices to simplify expressions with Maths Genie

See how to solve "exponential" equations using the laws of indices where the base numbers are equal here

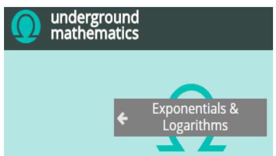
Now try the questions overleaf!

Learn what a logarithm is with this TED-ED talk

Click below to learn the Laws of Logarithms here



Now read – or better still, try this problem, of fitting the radius of a planetary orbit with the length of its year.



You need to **LEARN** and be able to **USE** the laws of indices. You should have met them on your HIGHER tier GCSE Maths course:

1. $x^a \times x^b = x^{a+b}$

2. $x^a \div x^b = \frac{x^a}{x^b} = x^{a-b}$ which also gives $x^0 = 1$

and $x^{-n} = \frac{1}{x^n}$ Remember: NEGATIVE powers mean RECIPROCALS

3. $(x^a)^b = x^{ab}$ which also gives $x^{\frac{1}{n}} = \sqrt[n]{x}$

and $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$ Remember: FRACTIONAL powers mean ROOTS

Example 1 Evaluate the following without a calculator:

a) 4^0 b) 4^{-3} c) $9^{\frac{1}{2}}$ d) $64^{\frac{1}{3}}$ e) $8^{\frac{2}{3}}$

a) $4^0 = 1$ b) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$ d) $9^{\frac{1}{2}} = \sqrt{9} = 3$ d) $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$ e) $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

Example 2 Evaluate the following without a calculator:

a) $16^{-\frac{1}{2}}$ b) $125^{-\frac{1}{3}}$

These all have indices that are both NEGATIVE (so are reciprocals) and FRACTIONS (so are roots). Deal with the negative sign in the index first:

a) $16^{-\frac{1}{2}} = \frac{1}{16^{1/2}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$ b) $125^{-\frac{1}{3}} = \frac{1}{125^{1/3}} = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$

Example 3 Evaluate the following:

a) $\left(\frac{4}{25}\right)^{-2}$ b) $\left(2\frac{2}{49}\right)^{\frac{1}{2}}$

a) $\left(\frac{4}{25}\right)^{-2} = \frac{1}{\left(\frac{4}{25}\right)^2} = \frac{1}{\left(\frac{4^2}{25^2}\right)} = \frac{1}{\left(\frac{16}{625}\right)} = \frac{625}{16}$ b) $\left(2\frac{2}{49}\right)^{\frac{1}{2}} = \left(\frac{100}{49}\right)^{\frac{1}{2}} = \sqrt{\frac{100}{49}} = \frac{\sqrt{100}}{\sqrt{49}} = \frac{10}{7}$

Example 4 Simplify the following expressions, re-writing each one using only <u>positive</u> indices:

a) $(2x)^3$ b) $\frac{2x^5}{x^3}$ c) $\frac{9x}{3x^5}$ d) $\frac{(2x)^3}{\sqrt{x}}$ e) $2\sqrt{x} \times (3x)^2$

a) $(2x)^3 = 2^3 \times x^3$ b) $\frac{2x^5}{x^3} = 2 \times \frac{x^5}{x^3} = 2 \times x^{5-3} = 2x^2$ c) $\frac{9x}{3x^5} = \frac{9}{3} \times \frac{x}{x^5} = 3x^{1-5} = 3x^{-4} = \frac{3}{x^4}$

Note: In part c), <u>**only the x**</u> is raised to the power negative 4. So <u>**only**</u> x moves to the bottom; the three stays on top.

d) $\frac{(2x)^3}{\sqrt{x}} = \frac{2^3 \times x^3}{x^{\frac{1}{2}}} = 8\frac{x^3}{x^{\frac{1}{2}}} = 8x^{3-\frac{1}{2}} = 8x^{2\frac{1}{2}} = 8x^{\frac{5}{2}}$ e) $2\sqrt{x} \times (3x)^2 = 2x^{\frac{1}{2}} \times 3^2 \times x^2 = 18x^{\frac{1}{2}+2} = 18x^{\frac{5}{2}}$

Now try these:

Please evaluate the following without a calculator, giving an exact answer:

1. 5^{-2} 2. $16^{\frac{1}{2}}$ 3. 7^0

4. 2^{-3} 5. $\left(\frac{9}{16}\right)^{\frac{1}{2}}$ 6. $8^{\frac{1}{3}}$

7. $4^{-\frac{1}{2}}$ 8. $\left(\frac{2}{3}\right)^{-2}$ 9. $(25)^{\frac{3}{2}}$

Please simplify the following expressions and re-write each of them as an expression using only positive indices where necessary:

10. $(3x)^3$ 11. $25x^{-3}$ 12. $3x^5 \times 5x^6$

13. $4x \times 3x^{-3}$ 14. $\frac{6x^5}{2x^3}$

15. $\frac{8x^3}{6x^7}$ 16. $\frac{(3x)^3}{6x}$

17. $\frac{\sqrt{x}}{x^2}$ 18. $\frac{4x^5}{2\sqrt{x}}$

19. $\frac{5x^2}{\sqrt[3]{x}}$ 20. $(2x)^3 \times 6\sqrt{x}$

Answers

1. $\frac{1}{25}$ 2. 4 3. 1 4. $\frac{1}{8}$ 5. $\frac{3}{4}$

6. 2 7. $\frac{1}{2}$ 8. $\frac{9}{4}$ 9. $5^3 = 125$ 10. $27x^3$

11. $\frac{25}{x^3}$ 12. $15x^{11}$ 13. $\frac{12}{x^2}$ 14. $3x^2$ 15. $\frac{4}{3x^4}$

16. $\frac{9x^2}{2}$ 17. $\frac{1}{x^{3/2}}$ or equivalent 18. $2x^{9/2}$ or equivalent

19. $5x^{5/3}$ or equivalent 20. $48x^{7/2}$ or equivalent

FINAL ASSIGNMENT

This final assignment is mainly a selection of Higher Tier GCSE questions, taken from past papers or specimen papers. You should complete all of them correctly, showing clear, logical workings. Please bring your work in when classes start in the Autumn. If you can print these pages off, please write your workings in the spaces provide. If you cannot print them off, please either just write your workings on ordinary A4 paper, or send us an email before July 1st and ask us to post a hard copy of these Final assignment pages to you.

Be sure to complete these questions before the A-level course starts. **Remember that help is available online should you need it**

By completing these questions correctly, you should have the essential skills need to start the course successfully. If you cannot print these pages, please write answers on lined paper.

1 Circle the expression that is equivalent to $(4a^5)^2$ **[1]**

$16a^{10}$ $\qquad$ $16a^7$ $\qquad$ $8a^{10}$ $\qquad$ $16a^7$

2 (*a*) Simplify $(p^2)^5$ **[1]**

(*b*) Simplify

$$\frac{12x^7y^3}{6x^3y}$$

[2]

3 Simplify a) $\frac{24x^2}{8\sqrt{x}}$ b) $x^3(\sqrt{x}+2)$ **[2]**

4 (a) Factorise **fully** $9y^3 - 6y$ **[2]**

(b) Factorise $3x^2 - 22x + 7$ **[2]**

5 Solve $5x^2 = 10x + 4$. Give your answers in exact ("surd") form. [**4**]

6 $2x^2 - 6x + 5$ can be written in the form $a(x - b)^2 + c$

where a, b and c are positive numbers.

Work out the values of a, b and c. **[3]**

7 The straight line $\mathbf{L}$ has equation $3y = 4x + 7$

The point A has coordinates $(3, -5)$

Find an equation of the straight line that is perpendicular to $\mathbf{L}$ and passes through A. **[3]**

8 Make m the subject of the formula

$$f = \frac{3m + 4}{m - 1}$$

[3]

9 Make *k* the subject of the formula $y = \sqrt{2m - k}$ **[2]**

10 Simplify

$$\frac{2x^2 - x - 28}{2x^3 + 7x^2}$$

[3]

11 A curve has equation $y = 4x^2 + 5x + 3$
A line has equation $y - x = 2$
Find the point of intersection between the curve and the line. **[4]**

12 Write

$$\frac{5}{x+1} + \frac{2}{3x}$$

as a single fraction in its simplest form. **[2]**

13 Solve

$$\frac{x}{4} - \frac{2x}{x+2} = 1$$

Give your solutions to 2 decimal places.
You **must** show your working. **[6]**