Mathematics

Welcome

Mathematics is one of the most widely taken A-level courses nationally, recognised by many for its “usefulness”.

This Headstart pack will give you a chance to practice and revise some essential skills from GCSE so that when you start the A level course here in September you are well prepared for the challenge!

**FAQ**

**What is the difference between Maths and Further Maths?**

The main difference is that Further Maths is a TWO A-level course, covering all of the A-level Maths course PLUS a whole extra A-level of Further Maths. This means having twice as many lessons and twice as much homework, so it is best for students who are pretty committed to studying Maths at university, or a very closely related subject like Engineering or Physics. Nationally, Further Maths is almost only taken by the strongest A-level Maths students, so we run AS Further Maths in the first year as a fourth subject, then if things go well, we can review your options at the end of year 1.

**How many lessons will I have each week?**

A-level Maths students will have three 1½ hour lessons each week, with an additional 25 minutes of academic tutorial support available for each class each week. Further Maths students will have another three lessons and academic tutorial support each week.

**How much self-study/homework will I get each week?**

Like all A-level course at Richard Huish, you should expect to do at least 4 hours each week if you are doing A-level Maths. Those choosing Further Maths would expect to do another 4 hours each week.

**What help is available if I get stuck?**

You will get stuck, and the Maths team recognise that this will happen. Once you have made a fair attempt to try and figure things out for yourself (this is what “resilience” in problem-solving looks like), your Maths teacher will be more than happy to help you.

**What grades should I have?**

The college’s recommended minimum to study A-level Maths is a grade 7 at GCSE in Mathematics in addition to the standard college entrance requirements. For Further Maths it is a grade 8, with a general GCSE profile strong enough to support doing 4 A-levels in the first instance. This is decided by a senior member of staff in each case when joining us in September.

**Do I have to take 4 A-levels to do Further Maths?**

Yes, you will take 4 A-levels in year 1.

You will sit an AS in Further Maths at the end of year 12.

After this you can choose whether to continue the course into year 2 to complete the full Further Maths A-level course.

**What will I need?**

* Your scientific calculator from GCSE for the first lesson.
* You will need a CASIO FX991CW, you cannot complete the course without one of these, so feel free to buy one over the summer.
* HOWEVER, bursary students will be able to use their bursary to purchase the calculator from college in September - so if you think that might apply to you - or you are in any way unsure please wait until September.

**What will I study?**

Here is an example of the sort of question you will be able to answer by the end of year 1:



**Getting ready…What you need to do……**

You need to complete the Compulsory Assignment which is on the last 6 pages.

**Please bring your completed work to your first maths lesson when we will check it**. If you want to print these pages off and write on them that’s fine, otherwise please just write your workings on ordinary A4 paper.

To help you check whether you have covered the skills you need to start the course smoothly, and whether you have any key gaps in your skill set you need to work through the six sections below.

Each section has links to online teaching resources which you can use to help you, followed by questions to complete along with answers. Work through as many of the resources as you need so that you feel confident to complete the questions.

Don’t forget there is a separate Headstart booklet for Further Maths with an additional assignment.

**Section 1: Indices**

At A-level, you will have to handle a wide range of expressions involving indices (also known as “powers”). You will also need to be able to use the laws of indices to simplify expressions.

[Now try this test from BBC Bitesize](https://www.bbc.co.uk/bitesize/guides/z826y4j/test)

Explore using the laws of indices at 3 different levels with this [Geogebra quiz](https://www.geogebra.org/m/FDFG95rd).

A recap of all the main laws of indices, including the meaning of negative and fractional powers with [Exam Solutions here](https://youtu.be/UQqJo5XchKA)

Start

Not sure?

Recap [positive integer indices here](https://www.youtube.com/watch?v=r_No-gMg6Fc&feature=emb_rel_end)

Recap the [multiplication law of indices here](https://youtu.be/g5V71lkxpQk)

Recap the [division law of indices here](https://youtu.be/wF8lugDwS_g)

Recap [negative integer indices here](https://youtu.be/SW9nb-13V6E)

Recap the [“power to a power” law here](https://youtu.be/yFb8VqLq9JY)

Recap [fractional indices here](https://youtu.be/AB3ARY9V0Cw)

See how to solve “exponential” equations using the laws of indices where the base numbers are equal [here](https://youtu.be/8P2bDYtlICo)

Now practice using the laws of indices to simplify expressions with [Maths Genie](https://www.mathsgenie.co.uk/questions-indices.html)

Now try the questions overleaf!

You need to **LEARN** and be able to **USE** the laws of indices. You should have met them on your HIGHER tier GCSE Maths course:

1. which also gives

 and Remember: NEGATIVE powers mean RECIPROCALS

1. which also gives

 and Remember: FRACTIONAL powers

 mean ROOTS

Example 1 Evaluate the following without a calculator:

 a) 40 b) 4-3 c) d) e)

a) 40 = 1 b) d) d) e)

Example 2 Evaluate the following without a calculator:

1. b)

These all have indices that are both NEGATIVE (so are reciprocals) and FRACTIONS (so are roots). Deal with the negative sign in the index first:

a) b)

Example 3 Evaluate the following:

 a) b)

a) b)

Example 4 Simplify the following expressions, re-writing each one using only positive indices:

 a) b) c) d) e)

a) b) c)

Note: In part c), **only the**  is raised to the power negative 4. So **only** moves to the bottom; the three stays on top.

d) e)

**Now try these (this should take 30 mins):**

Please evaluate the following without a calculator, giving an exact answer:

1. 5 – 2  2. 3. 70

4. 2 – 3  5. 6.

7. 8. 9.

Please simplify the following expressions and re-write each of them as an expression using only positive indices where necessary:

10. (3*x*)3 11. 12. 3*x*5 x 5*x*6

13. 4*x* x 3*x* – 3 14.

15. 16.

17. 18.

19. 20.

**Answers**

1. 2. 4 3. 1 4. 5.

6. 2 7. 8. 9. 10. 27x3

11. 12. 15x11 13. 14. 3x2 15.

16. 17. or equivalent 18. 2x9/2 or equivalent

19. 5x5/3 or equivalent 20. 48x7/2 or equivalent

**Section 2: Factorizing**

Factorising algebraic expressions is so frequent in A-level Mathematics, that you will need to be very confident about doing it. The online videos linked below show you some common methods for doing this, though there are others.

Mathematicians use “by inspection” – which is “Maths-speak” for “Guess until you get it right, and then check that it *is* right!”

Factorising Quadratics (all forms)

[Try these factorising questions from Maths Genie](https://www.mathsgenie.co.uk/questions-factorising-harder-quadratics.html)

Factorising Quadratics (all forms)

Maths Genie lessons on factorising “trinomial” (3 term) quadratic expressions

[By the “sum/product” or “grouping” method](https://corbettmaths.com/2019/03/26/splitting-the-middle-term/)

[By the “difference of two squares” formula](https://youtu.be/IqN8Z1-nlsY)

[By “inspection” (the “professionals” way!)](https://www.youtube.com/watch?v=DvkCijZIRog&feature=emb_rel_pause)

Start

Revise the “difference of two squares” formula

[with this Corbett Maths video](https://corbettmaths.com/2013/02/08/difference-between-two-squares/)

Revise expanding and factorising two brackets

[Corbett Maths: Expanding Brackets](https://corbettmaths.com/2013/12/23/expanding-two-brackets-video-14/)

[Factorising “trinomials” like](https://corbettmaths.com/2013/02/06/factorising-quadratics-1/)

Revise factorising harder quadratics

[“Splitting the middle term” method (Corbett)](https://corbettmaths.com/2019/03/26/splitting-the-middle-term/)

[By “inspection” (the “professionals” way!)](https://www.youtube.com/watch?v=DvkCijZIRog&feature=emb_rel_pause)

Not sure? Maths Genie lessons on:

[Expanding one bracket](https://www.youtube.com/watch?v=0CoYV7CoumI&feature=emb_logo)

[Factorising with Common Factors (1 bracket)](https://www.youtube.com/watch?time_continue=2&v=VLDo8ifFO_8&feature=emb_logo)

[Now try these online factorisation questions](https://www.mathsgenie.co.uk/questions-factorising.html)

Now try the questions overleaf!

Some further tips on how to figure out the factors of a quadratic (but don’t forget common factors and difference of two squares for two term quadratics!)

Multiplication of brackets is carried out as follows:

D

C

B

A

D

A

(3*x* + 2)(*x* + 4) = 3*x* x *x* + 2 x *x* + 3*x* x 4 + 2 x 4

8 is the constant term and comes from the “product” (i.e. multiplication) of the last terms in each bracket (D)

B

 = 3*x*2 + 2*x* + 12*x* + 8

C

 = 3*x*2 + 14*x* + 8

The 3*x*2 term comes from the product of the **first terms in each bracket** (A)

The 14*x* term bracket (D) comes from the product of the **outer terms plus the product of the inner terms** [(B) and (C)]

To factorise 3*x*2 + 14*x* + 8 into two brackets, first note that:

+3*x*2 comes from the product (i.e. multiplication) of the **first terms in each bracket** (A)

+14*x* comes from the product of the **outer terms plus the product of the inner terms** [(B) and (C)]

+8 is the product of the **last terms in each bracke**t (D)

Also note that:

* If the x term is positive and the constant term is positive, then both brackets will contain “+”
* If the x term is negative but the constant term is positive then both brackets will contain “–”.
* Finally if the constant term is negative, regardless of the x term the brackets will contain one “+” and one “–”.

**Now try these (this should take 20 mins):**

Factorise fully each of these quadratics. (By factorise fully, we mean that you take out the highest common factors, both numerical and algebraic).

1. 2.

3. 4.

5. 6.

7. 8.

Answers:

1. 2. 3. 4.

5. 6. 7. 8.

**Section 3: Quadratics, including completing the square**

Quadratic equations are used very widely throughout the A-level Maths course from the very beginning. You could be asked to solve them, sketch (*not* plot) their graphs or to “complete the square” and to use that form to find the maximum or minimum. Follow this map to revise most of these GCSE methods.

[Take this Geogebra quiz on solving quadratics by factorisation](https://www.geogebra.org/m/tKAZnWjn)

Optional: Revise how to write

 where

[Exam Solutions Video](https://youtu.be/kdCSN7NiB_E)

Optional: Learn how to **prove** the Quadratic Formula using the “Completed Square” form [here](https://youtu.be/A-JRFcexB78)

Revise solving quadratics using the quadratic formula with [this Exam Solutions video](https://youtu.be/u51mFZ-1HZQ)

Revise solving a quadratic equation by factorisation.

[Exam Solutions: Solution by factorising](https://www.examsolutions.net/gcse-maths/algebra/algebra-quadratic-equations/#quadraticequations1)

Start

Revise how to write

 where

[“Completing the Square” with Exam Solutions](https://youtu.be/om8eNP1YV4c)

Not sure?

A shorter [video from Corbett Maths](https://youtu.be/wJ_tLEwEEi8) showing how to solve a quadratic by factorisation

Try some of [these self-test questions](https://www.mathsgenie.co.uk/questions-solving-quadratics-factorising.html) on Solving Quadratics from BBC Bitesize

Optional [self-test questions](https://www.mathsgenie.co.uk/questions-completing-the-square.html) on Completing the Square from Maths Genie

Optional [self-test questions](https://www.mathsgenie.co.uk/questions-quadratic-formula.html) on solving a quadratic using the formula

Now try the questions overleaf!

Note that to solve any quadratic, you **must** **first** begin by re-writing it in the form:

(i.e. “…”) where , and are some fixed numbers, and is not zero.

Note:

* If , then the equation looks like . If is negative, use “difference of two squares”.
* If the equation looks like . These can be solved using “common factorization” like this:

There are two methods of solving quadratics at GCSE which we use extensively at A-level:

* Factorization

Write as two brackets or “factors” , and setting each bracket equal to zero to solve.

If the quadratic is a simpler case where , we get common factorization, and just one “bracket”

* Using the formula

If then

Although many calculators will solve quadratics, **you are still expected to be able to do it “by hand”**. The skill of “factorizing” in the first method above is especially useful, and A-level Maths students do this very quickly.

**Now try solving these equations (this should take 40 mins)**

1. *x*2 + 7*x* + 12 = 0 2. 3*x*2 + 5*x* −2 = 0

3. *x*2 − 2*x* + 1 = 0 4. 3*x*2 + 5*x* + 2 = 0

5. *x*2 − 1 = 0 6. *x*2 − 2*x* − 3 = 0

7. 8*x*2 + 14*x* + 3 = 0 8. *x*2 − 4 = 0

9. 4*x*2 − 9 = 0 10. 12*x*2 − 5*x* = 3

Please solve the following quadratics using the formula, giving your answer in simplest surd form:

11. *x*2 − 3*x* − 2 = 0 12. 4*x*2 − 4*x* − 1 = 0

13. 3*x*2 + 2*x* − 7 = 0 14. 7*x*2 + 9*x* + 1 = 0

Answers

1. (x + 4)(x + 3) = 0; x = -4 or -3 2. (3x – 1)(x + 2) = 0; x = or -2 3. (x – 1)(x – 1) = 0; x = 1 (or 1)

4. (3x + 2)(x + 1) = 0; x = or -1 5. (x + 1)(x – 1) = 0; x = -1 or +1 6. (x + 1)(x – 3) = 0; x = -1 or 3

7. (4x + 1)(2x + 3) = 0; x = or 8. (x + 2)(x – 2) = 0; x = -2 or +2 9. (2x + 3)(2x – 3) = 0; or

10. (3x + 1)(4x – 3) = 0; x = or

11.

12.

13.

14.

**Section 4: Straight line coordinate geometry**

There are many types of graph that you will study, sketch and interpret during the A-level Maths course. The most basic is that of the straight line. At GCSE its equation is usually in the form where is the gradient and the intercept on the axis.

At A-level, we frequently rearrange this into other, equivalent forms like where , and are constant (“fixed”) numbers. You will need to be able to rearrange this into whenever necessary. You will also need to be able to find the equation of a straight line given either

* two points on the line, or
* one point (NOT necessarily the intercept!) and the gradient of the line.

Other knowledge you will also need include:

* recognising when two lines are parallel or perpendicular from their gradients alone,
* finding the distance between two points
* finding the coordinates of the midpoint of a line (segment).

[Check your knowledge with this quiz from Bitesize](https://www.bbc.co.uk/bitesize/guides/z9387p3/test)

Recognising and using Parallel lines:

[Examples from Corbett Maths](https://corbettmaths.com/2013/06/06/graphs-parallel-lines/)

Recognising and using Perpendicular lines:

[Explanation and examples from Corbett Maths](https://youtu.be/PrwhdgnLK5k)

Revise how to:

Use Pythagoras’s Theorem to find the distance

between two points with [Exam Solutions here](https://youtu.be/q3XS1m1TVeQ)

Find the coordinates of the midpoint of a line [here with Exam Solutions](https://youtu.be/WVFTQEVhs_E)

Revise the equation of a line given the gradient and any point with [Exam Solutions here](https://www.youtube.com/watch?v=2aKmbbfMzF4&feature=youtu.be)

More from Exam Solutions [here](https://youtu.be/oqzURxhYuTU)

Start

Explore the effect and meaning of “” and “” in the with [this Geogebra app](https://www.geogebra.org/m/ueBveY4g#material/PHDPqdWh)

Move the blue points around and change the gradient and intercept to fit the line through the points in this [Geogebra app](https://www.geogebra.org/m/ueBveY4g#material/sRNKN3EH)

Learn how to find the equation given any two points with [Corbett Maths here](https://youtu.be/MJKPASvp0qY)

How to find the equation given the gradient and any one point with [Exam Solutions](https://youtu.be/2aKmbbfMzF4)

Not sure? Remind yourself of what the gradient means [here with Exam Solutions](https://www.examsolutions.net/gcse-maths/algebra/algebra-gradient/#gradient1)

 [Here are some practice questions](https://www.mathsgenie.co.uk/questions-gradient.html)

if you want them

Now try the questions overleaf!

**Now try these questions**

This should take 1 hour.

1. Find the gradient of the line through each pair of points:

 a) and b) and

2. Calculate the exact\* distance between each pair of points.

(\*Exact means leave your answer in surd form)

 a) and b) and

3. Find the coordinates of the midpoint of each pair of points:

 a) and b) and

4. Work out the gradient and the -intercept of these lines:

 a) b)

5. Find the equation of the line through each pair of points:

 a) and b) and

6. The line has equation

 a) Find the equation of the line parallel to and which passes through the point

 b) Find the equation of the line perpendicular to and which passes through the point

7. The line is shown on the grid.

 Find an equation for .

8. is the point with coordinates . is the point with coordinates .

 The gradient of the line is . Work out the value of .

Answers.

All answers are given in the simplest (“cancelled down”) exact form. Decimals are **only** given as an alternative, where they are exact equivalent of the fractional value.

1 a) b) -2 2 a) b) 3 a) or b)

4 a) Gradient is -intercept is b) Gradient is -intercept is

5 a) b)

6 a) or b) or

7. 8.

**Section 5: Algebraic Manipulation, including Changing the subject and algebraic fractions**

Rearranging algebraic expressions is *the* basic skill used by all mathematics students at A-level to complete a solution to any problem, or to prove any formula or result that has to be used.

The following resources give examples of the kinds of GCSE techniques that you will be using frequently from the very start of the course. You will add new techniques to these as the course progresses.

Now see how adding algebraic fractions is similar to adding numerical fractions.

[Video showing how to add two fractions with different denominators](https://www.bing.com/videos/search?q=video+on+adding+fractions&docid=608016770119369960&mid=DC3CFC1DBFF5B2351A30DC3CFC1DBFF5B2351A30&view=detail&FORM=VIRE)

[Maths Genie video lesson on simplifying algebraic fractions](https://www.youtube.com/watch?time_continue=171&v=6DHOXIaPF7g&feature=emb_logo)

Start

Revise Rearranging Formulae

[Maths Genie Lesson Changing the Subject of a Formula (Higher)](https://www.youtube.com/watch?v=d7_IGWBu66o&feature=emb_logo)

Not sure?

[Watch this Maths Genie Lesson on Changing the Subject of a Formula (Foundation)](https://www.mathsgenie.co.uk/changing-the-subject1.html)

[Here are some more examples from Corbett Maths](https://www.youtube.com/watch?v=8U9u_itcs7k&feature=emb_rel_pause)

Now try the questions overleaf!

Revise Simplifying Algebraic Fractions

[Exam Solutions lesson on simplifying algebraic fractions by factorising first and cancelling](https://www.examsolutions.net/gcse-maths/algebra/algebra-rational-expressions/#rationalexpressions1)

[Maths Genie video on how to add or subtract algebraic fractions using a common denominator](https://www.youtube.com/watch?time_continue=7&v=khZnlxI6MSw&feature=emb_logo)

[Now try these algebraic fractions questions from Maths Genie](https://www.mathsgenie.co.uk/questions-algebraic-fractions.html)

Work each one out first, then check your answer.

Not sure? Start with simplifying some numerical examples

[Corbett Maths lesson on simplifying numerical fractions by dividing top and bottom by a common factor](https://corbettmaths.com/2013/03/03/simplifying-fractions-2/)

[Maths Genie video lesson on simplifying algebraic fractions](https://www.youtube.com/watch?time_continue=171&v=6DHOXIaPF7g&feature=emb_logo)

**Now try these (this should take 20 mins):**

1. Rearrange each of these formulae to make the subject

 a)

 b)

 c) (Hint: Get terms on one side and factorise)

 d) (Hint: Multiply out bracket, get terms on one side and factorise)

 e) (Hint: Square root both sides first – remember to write !)

2. (a) Simplify

 (b) Make the subject of the formula

3. Simplify fully

4. Solve

 You **must** show your working.

Answers.

Note that there are often several possible ways to write a correct answer. Some equivalent options are shown here:

1 a) or b) or c) or

 d) or e) or (Other possible answers exist)

2 a) b) 3 4.

**Section 6: Simultaneous equations**

Look at these two objects: Surprisingly, mathematicians see these two objects as the *same thing*!

The equation, if “understood” correctly, describes the straight-line graph. This means that a great many problems in coordinate geometry can be solved using the algebraic equivalent. This is what you do when solving simultaneous equations at GCSE.

Click below to revise solving simultaneous equations with other non-linear equations



Click below to revise how to find the points of intersection of a line and a circle



Click the picture to revise solving two linear equations by elimination



Start

Not sure?

Click below for a selection of solving two linear simultaneous equations from Exam Solution



Try some simultaneous self-test questions from [Maths Genie](https://www.mathsgenie.co.uk/questions-quadratic-simultaneous.html)

Try [these self test questions on Maths Genie](https://www.mathsgenie.co.uk/questions-simultaneous.html)

Now try the questions overleaf!







For two linear equations, there are two methods using algebra that can be used:

Method 1: (“Elimination”) and Method 2: (“Substitution”)

 E.g. Solve (equation 1)

 and (equation 2)

**Method 1:** **Method 2:**

(equation 1) : Rearrange equation 1 to make the “subject”:

 (3) (subtract from both sides)

(equation 2) : Substitute (“replace”) in equation 2 with :

 (4) (Note use of brackets!)

(eq. 3 + eq. 4): (Expand the bracket)

Now find : (Subtract , add )

(equation 1)

 Now find :

**Now try these (this should take 20 mins):**

Solve the following pairs of linear simultaneous equations, giving your answers in their simplest form where necessary. You may use either method, but must write out full workings:

1. 2.

3. 4.

**Answers:**

1. 2. 3. 4.

For one linear equation, and one quadratic, you MUST use substitution. Rearrange the LINEAR equation to make either or the subject, then substitute (i.e. “replace”) it in the quadratic equation.

Example:

Solve and

Rearrange the linear equation (the first one here), to make the subject. (It is easier to make the subject):

 (subtract from both sides)

Substitute for in the second quadratic equation. **Use brackets**:

Expand the bracket carefully. Remember that :

It’s a LOT easier to divide both sides by the highest common factor of 12 in these terms:

Use factorisation or the formula to solve:

So or

Now use to find the values:

When , When ,

**Now try these (this should take 20 mins):**

1. Solve and

2. Solve and

3. Solve and

4. Solve and

Answers

1. and 2. and

3. and 4. only

**Compulsory Assignment to be handed to your teacher in your first maths lesson:**

Complete the questions without a calculator and **showing full workings**. (This task should take up to 1 hour.)

**Section 1: Indices**

**1** Circle the expression that is equivalent to **[1]**

**2** (*a*)Simplify **[1]**

 (*b*)Simplify

 **[2]**

1. Simplify a) b) **[2]**

**Section 2: Factorizing**

**4** (a) Factorise **fully** **[2]**

 (b) Factorise **[2]**

**Section 3: Quadratics, including completing the square**

**5** Solve . Give your answers in exact (“surd”) form. [**4**]

**6** can be written in the form

 where , and are positive numbers.

 Work out the values of , and . **[3]**

**Section 4: Straight-line coordinate Geometry**

**7** The straight line has equation

 The point has coordinates

 Find an equation of the straight line that is perpendicular to and passes through . **[3]**

**Section 5: Algebraic manipulation**

**8** Make the subject of the formula

 **[3]**

**9** Make *k* the subject of the formula **[2]**

**10** Simplify

 **[3]**

**11** Write

 as a single fraction in its simplest form. **[2]**

**12** Solve

 Give your solutions to 2 decimal places.

 You **must** show your working. **[6]**

**Section 6: Simultaneous equations**

**13** A curve has equation

 A line has equation

 Find the point of intersection between the curve and the line. **[4]**